Known unknowns, conceptual spaces
and the numerical aesthetic;
art and mathematics searching for a singular truth?

An essay by Andy Bullock

“What I admire most about your art is its universality” said Einstein.

“You don’t say a word and yet the world loves you.”

“It’s true” replied Chaplin

“but your fame is greater and yet nobody in this crowd even understands you!”.

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Abstract

As exemplified by the opening quotation from an exchange between Albert Einstein and Charlie Chaplin, at the opening of the movie ‘City Lights’, this essay aims to investigate the attraction and gravitational pull of the naturally *unknown*, of how we can be drawn to, and fascinated by, that which we do not understand; making the connection in purpose of creative thinking between the conceptual artist and the mathematician, especially in relation to the critical and creative investigation of theoretical and abstract problems concerned with *conceptual spaces;* and is framed within the context of my own personal issues with numbers and numerical logic - living as I do with a *Specific Learning Disability* (SLD) now widely known as *Dyscalculia* I will also endeavour to explain how this informs and inspires my own fine-art practice.

Introduction

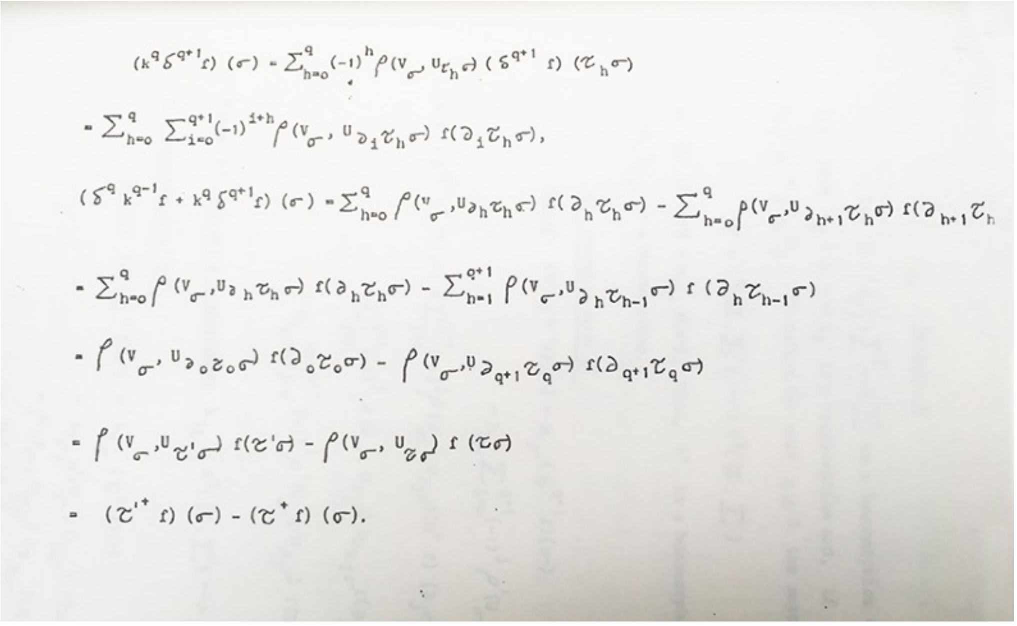
In 2015 I became custodian of the extensive archive of Clifford Hugh Dowker (1912 - 1982), an eminent Canadian-born mathematician who had a significant impact on developmental thinking in the mathematical discipline known as topology. In particular it was within the field of Sheaf Theory[1] and Knot Theory[2] that he left his deepest mark, the latter of which he is most notably known for *Dowker Notation*[3]*,* the theory of which was first published in ‘*Classification of Knot Projections’* (Dowker/Thistlethwaite, 1983). It was on first opening the hard-bound copy of ‘*Lectures on Sheaf Theory’* (Dowker, C. 1956), a weighty, quarto-sized 230 page book with its seriously-academic dark-blue cover and embossed silver-foiled typographic title that my innate curiosity was awakened.

[1] “A special mathematical tool which provides a unified approach to establishing connections between local and global properties of topological spaces (in particular geometric objects) …” Springer Encyclopaedia of Mathematics.

[2] The study of mathematical knots.

[3] A computational method of representing knots numerically.

And it was only when I *physically* handled this book, this *tome*, for the first time that I could feel the literal *intellectual weight* of the contents, the enormity of *effort* imbued in all the complex mathematical symbols, added to the text by (Dowker's own) hand *(Fig.1),* that I began this voyage of curiosity; my very own journey into an (ultimately for me) *unknowable world* that would continue to inspire me creatively and artistically in ways that I have barely yet begun to realise.



$$(k^q \delta^{q+1} f)(\sigma) = \sum_{h=0}^{q} (-1)^h \rho(V_\sigma, U_{\tau_h \sigma}) (\delta^{q+1} f)(\tau_h \sigma)$$

$$= \sum_{h=0}^{q} \sum_{i=0}^{q+1} (-1)^{i+h} \rho(V_\sigma, U_{\partial_i \tau_h \sigma}) f(\partial_i \tau_h \sigma),$$

$$(\delta^q k^{q-1} f + k^q \delta^{q+1} f)(\sigma) = \sum_{h=0}^{q} \rho(V_\sigma, U_{\partial_h \tau_h \sigma}) f(\partial_h \tau_h \sigma) - \sum_{h=0}^{q} \rho(V_\sigma, U_{\partial_{h+1} \tau_h \sigma}) f(\partial_{h+1} \tau_h$$

$$= \sum_{h=0}^{q} \rho(V_\sigma, U_{\partial_h \tau_h \sigma}) f(\partial_h \tau_h \sigma) - \sum_{h=1}^{q+1} \rho(V_\sigma, U_{\partial_h \tau_{h-1} \sigma}) f(\partial_h \tau_{h-1} \sigma)$$

$$= \rho(V_\sigma, U_{\partial_0 \tau_0 \sigma}) f(\partial_0 \tau_0 \sigma) - \rho(V_\sigma, U_{\partial_{q+1} \tau_q \sigma}) f(\partial_{q+1} \tau_q \sigma)$$

$$= \rho(V_\sigma, U_{\tau' \sigma}) f(\tau' \sigma) - \rho(V_\sigma, U_{\tau \sigma}) f(\tau \sigma)$$

$$= (\tau'^{+} f)(\sigma) - (\tau^{+} f)(\sigma).$$

Fig.1 A page of handwritten equation by Hugh Clifford Dowker, 'Lectures on sheaf theory' 1956

Curiosity - a very human desire to *know the unknown?*

Curiosity may very well have killed the cat but it is, and always has been, the foundational lifeblood of knowledge-acquisition for the human race; the force that drives us to evolve and develop as a species. It is this search for the *currently unknown* that drives conceptual artists and mathematicians to push the boundaries of creative and critical investigation to previously unexplored domains, with no prior notion of what the outcome may be. And, just as the early neanderthal man who first ventured beyond the edge of his village and over the most distant hill, and the fearless, seafaring explorers of the middle ages who set sail for the distant horizon understood, there is no desire greater than that of seeking to find and understand that which is, at the moment, *unknown;* which at the time is actually *unknowable* and perhaps even *beyond imagination*. One of the most dramatic evocations of this notion in modern times must surely be that of the astronauts who locked themselves into an Apollo rocket capsule at Kennedy Space Centre, Merritt Island, Florida, said their prayers, crossed their fingers, made their peace with whatever or whomever before being blasted to 25,000mph to escape the Earth's gravity, to penetrate the unknown; *the beyond*.

I believe it is this fundamental human desire to *know the unknown* that engenders a similarity of creative thought-process in both conceptual artists and theoretical mathematicians working in the more esoteric areas such as abstract, algebraic topology. I aim to explore this perceived similarity of thought-process and motivation and attempt to make the link in *creative purpose* in the philosophical approaches of these, on the face of it, wildly different practitioners; and, from a very personal point of view, as someone who has a defined learning disability with numbers and numerical problems known as *Dyscalculia*, I will examine my own motivations and attempt to explain why I choose to use the work of abstract mathematicians as inspiration and a launch-point into developing a body of work exploring these concepts from a fine-art perspective; this therefore is a very personal voyage into my own idea of *the beyond.*

Dyscalculia - dyslexia's lesser-known cousin

Dyscalculia was first recognised in 1919 by Salomon Henschen (1840-1927), a Swedish neurologist who found it was possible for a person to have an impaired mathematical ability that bore no relation to general intelligence. As a rather interesting aside it was Henschen that administered to Lenin after his series of strokes and it was also Henschen along with his son, Folke Henschen (1881-1977), who carried out an autopsy of Lenin's brain (Witztum, Ely & Lerner, Vladimir, 2002). I digress; it was only as recently as 1974 when Czech psychology researcher Ladislav Kosc (1924-2016) described this condition as a “structural disorder of mathematical abilities” (Kosc. L, 1974) that Dyscalculia (from the Greek and Latin meaning literally ‘counting badly’) became recognised as a *Specific Learning Disability.* However it wasn’t until significantly later that Dyscalculia became sufficiently accepted to be considered seriously by the educational establishment and recognised as a possible problem for some students; it is now widely believed that Dyscalculia effects 3-6% of the population (Butterworth, B. Yeo, D. 2004). This lack of knowledge of the disorder within the educational establishment at the time is all too well-known to me. Although achieving consistently high grades at school in the humanities subjects, no matter how hard I worked I consistently achieved the lowest of grades in Mathematics, Physics and Chemistry. Despite concerted effort on my part I managed to score an ‘ungraded’ (below 18%) result in my maths ‘O’ level (GCSE); this was absolutely devastating, to say the least, for an already less than confident 16 year old. I attended a well-respected Grammar School which was largely modelled on a traditional English *public* school (masters in gowns and mortar boards) and its students were generally expected to be higher than average academic achievers. The memories are still vivid in my mind when, on more occasions than I care to remember, if I ever questioned the maths master about something or asked for an explanation or simply said “I am sorry sir, but I just don’t understand this”, the response would be a swift whack around the head (yes, teachers really did that in the early seventies) and told “Boy, you must be stupid”. The emotional effect that treatment can have on a young person who is *seriously* trying to work and understand cannot be underestimated and this level of institutional ignorance from the educational establishment can seriously impact a young person’s life for many years; however, times, thankfully have changed and Dyscalculia is now taken very seriously indeed by schools.

And so, this is the background and context within which I approach this subject of higher, abstract, *conceptual* mathematics; as someone who could not even pass a maths ‘O’ level due to an (at the time) undiagnosed and *unrecognised* Specific Learning Disability. I firmly believe it is my *innate ignorance* in this area that allows me to see the beauty and find what I perceive as the *imagined poetics* in the impenetrable and opaque language of abstract equation; for it is most certainly not *just about numbers* as we will see in the following pages.

Of topology, knots and *conceptual spaces*

Topology is the branch of mathematics that investigates the properties of *spacial forms*. It can be used to describe the parameters of a geometric object under a continuous *morphing* and *reforming* process which could include twisting, stretching, crumpling, bending and general deformation. It can, for example, be used to describe *mathematically* the mind-bending properties of the notoriously baffling *Möbius Strip (Fig.2)* which, rather magically to my mind, has only *one* surface and *one* edge as you would discover if you ran a finger along its curves; and it looks like this -

Fig.2 Mobius strip

At its most basic, topology can explain how a material's shape can be completely *deformed* and *reformed* into a new shape without losing its core properties - hence the (possibly *only* topological) joke regarding a topologist not knowing the difference between a doughnut and a coffee mug. As improbable as this sounds from a practical perspective, it is actually possible using the maths of topology to prove this seemingly impossible transformation; in these terms the doughnut and the coffee mug are said to be *topologically equivalent (Fig.3).*

Fig.3 A mug to a doughnut - a graphic example of topological equivalence

This is a much-used and very simplistic example that even I can understand, it being explained graphically in such an unscientific way. However a mathematician in this field does not need to show this *physically,* as the language of topology is not visual, they would describe it with numbers and symbols, using the *mathematical language* of topology; and that is precisely where my understanding ends and my *fascination* begins.

In a much broader sense topology investigates and explains the mathematics of *dimensional space*; and as these spaces often only exist in the minds of those who work in this area we can refer to them as *conceptual spaces.* And conceptual spaces can, by dint of pure definition, exist solely in conceptual thought without ever having to be realised *visually.* It is this ability of mathematicians working with such abstract notions in numerical equations to hold the *feeling* and *form* of their equations in their heads (as surely they must do) that interests me so much. It is my belief that the *construction* of these metaphysical spaces conjured, as if by magic, out of pages of impenetrable algebraic equations requires the application of a level of creativity and *vision* that is arguably synonymous with that applied by artists to the creation of *conceptual* art. The problem of modelling representations to explain these purely mathematical notions in a more graphic way has long vexed those wishing to simplify the subject for non-specialists and to convey the information in a way that does not rely simply on numbers and symbols. As argued by Peter Gärdenfors in *'Conceptual Spaces'* (2000), " .. the mechanisms of concept acquisition, cannot be given a satisfactory treatment in these (purely numerical) representational forms. Concept learning is closely tied to similarity …. which is problematic for the symbolic and socialistic approaches." (Gardenfors, 2000). I completely concur with this and the only hope I would ever have of increasing my understanding in these fields would be through the use of different forms of *physical* presentation; and indeed Gärdenfors also proposes graphical *geometrical* structures in his explanation of the foundations of conceptual spaces and topological sets; one notably simple example is one that he uses to explain the human perception of taste, which is generally accepted to be made up of salt, sour, sweet and bitter; and using this example "… the quality space representing taste could be described as a four-dimensional space, a tetrahedron" *(fig.4)* (Gardenfors, 2000).

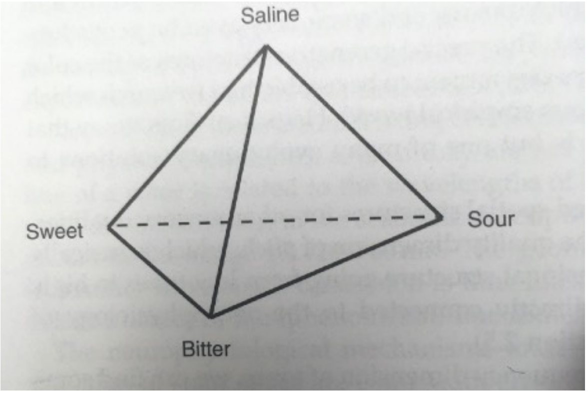


Fig.4 A tetrahedron as a conceptual space relating to descriptors of tastes

This therefore creates a *conceptual space* and it would be possible to give any *taste* a mathematical, numerical value based upon the point on the planes of the tetrahedron one placed the value of each *element* of the overall taste.

However I fear that this will only ever take this artist so far in the understanding of the topic and is perhaps the only way to achieve even a glimmer of understanding, albeit at a mere *scratching the surface* level for those without advanced mathematical knowledge and understanding. It is at the "*conceptual* level that most scientific and mathematical theorising takes place" (Gärdenfors, 2000) as graphic representations are simply not able to explain the complexities of abstract mathematical process and theory. There is however one branch of topology that often starts from the basis of a graphical representation and that is knot theory - the study of *conceptual* knots that can actually be drawn and notated in a graphic way although their mathematical notation and subsequent use and application *theoretically* rather quickly becomes highly abstract and conceptual in itself. Essentially knot theory concerns itself with the computation of hypothetical *endless* knots; for example, unlike a shoelace which has two *loose* ends, a knot used for these mathematical purposes has no ends and creates a *closed loop*. This closed loop can take many forms depending on how many over or under crossing points are involved; from the simplicity of a regular circle (which is rather poetically known as an *unknot)* to highly intricate and barely decipherable models. They can become highly complex as *structures* in themselves and it is this complex representation of knots that Dowker made such a significant part of his work. By devising a mathematical method of recording the various *crossover points* in any given knot *(Fig.5)* he proposed a theory for *knot classification*, in conjunction with Morwen Thistlethwaite (*Classification of Knot Projections,*1983), that enables a knot to be described *without* a graphic diagram thus making it useable in a purely abstract, computational and *conceptual* mathematical space. This renders all knots, regardless of complexity, to a string of numbers that, once one understands the method, can be decoded and used in abstract equations and computations.

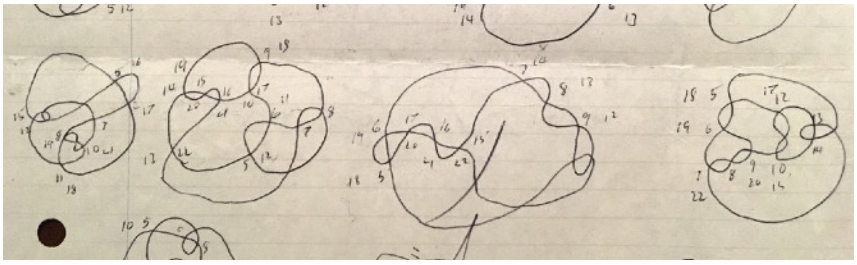


Fig.5 From Dowker's notebook with hand-drawn computational knot illustrations

Mathematicians using art and artists using mathematics

There are mathematicians who themselves are using 'art' as a physical descriptor for explanation of their work - I use the inverted commas here advisedly as whether or not one considers a 3D-printed model or a machine-produced 2D rendering 'art' is clearly open to scrutiny and the subject of a much broader and wide-ranging debate than being considered by this essay. "The language of mathematics is often less accessible than the language of art, but I can try to translate from one to the other, producing a picture or sculpture that expresses a mathematical idea." (Segerman, 2000). This quote from Henry Segerman to my mind confirms that the mathematics/art - art/mathematics link is a two-way street with the intellectual traffic moving freely in each direction along this *super-highway* of ideas. He does indeed use 3D printing to produce models to explain complex mathematical concepts to impressive effect; they are also by their very nature visually compelling, machine-produced to an immaculate finish and often defying the mind to work out quite how they are constructed. Interestingly, Segerman finds these techniques particularly well-suited to explaining the complexities and concepts pertaining to topological subjects such as conceptual knots, something that is finding its way into my own practice due to the exposure to the work of Dowker which originally inspired and underpins this whole investigation.

One could argue that using mathematics in this way, as a *generator* of art, almost makes the mathematics itself become the *artist responsible* for the resulting object, sculpture, drawing. It can also be seen perhaps more obviously as a *creative source* for inspiration in the work of avant-garde musicians such as Phillip Glass, Steve Reich or Michael Nyman all of whom could be broadly catalogued under the *minimalist* label, often using, as they do, sparse *mathematically-derived* patterns to inform and create their experimental compositions. And we should also perhaps note here (pun unavoidable) that in fact music itself exists in a form of *conceptual space*; the space being expressed and defined by the musical *key* it is written in and so the written, notated music itself, the *instruction*, of how to perform or recreate it becomes a form of *equation* with the *answer* being the resulting sound of the music we hear. Glass himself wrote at length about this *music by numbers* philosophy and approach in the extensive notes issued with the recording of his now legendary and groundbreaking operatic work *'Einstein on the Beach'* (Glass, Wilson,1989). Interestingly his explanations for the compositions often take the form of diagrams and numbers *(Fig.6)*, and not just *traditional* music manuscript scores, which renders them visually very similar to a mathematician's equations. "When numbers are used, they represent the rhythmic structure of the music. When *solfege* [4] is used, the syllables represent the pitch structure of the music. In either case, the text is not secondary,
but is a description of the music itself" (Glass, 1989).

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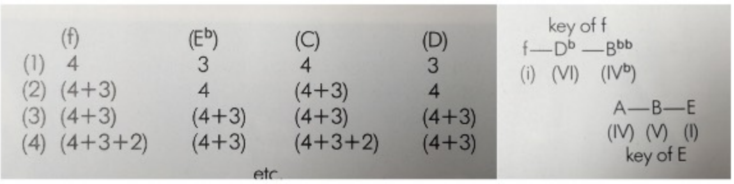
[4] Solfege refers to the system of describing musical notes as phonetic sounds such as 'do' 'ra' 'mi' 'so' fa'



	(f)	(E^{b})	(C)	(D)
(1)	4	3	4	3
(2)	(4+3)	4	(4+3)	4
(3)	(4+3)	(4+3)	(4+3)	(4+3)
(4)	(4+3+2)	(4+3)	(4+3+2)	(4+3)

etc.

key of f

$f — D^{b} — B^{bb}$

(i) (VI) (IV^{b})

A—B—E

(IV) (V) (I)

key of E

Fig.6 Equational instructions to musicians from the opera 'Einstein on the beach' 1976

The language Glass uses here to describe his music, his *equation*, is absolutely analogous with the way a mathematician would explain the use of symbols in *their* equational language; the semiotics used perform the same function in each discipline, to guide the reader to the same *conclusion* as the author.

Mathematical aesthetics and the elegance of numbers

There are, however, mathematicians who actually see the *beauty in the numbers* for the sake of the equations themselves. “Mathematics is the most beautiful and most powerful creation of the human spirit” wrote Stefan Banach, a Polish mathematician, widely regarded as one of the 20th century's most influential and important, and a man who clearly saw the benefit in bringing the artistic and poetic into the description of his mathematical practice. Noted English mathematician G.H. Hardy (1877 - 1947) embraced the language of poetics even further when he was quoted as saying “The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics”. (Hardy, 1967). Although I suspect we can assume these views were formed from the mathematicians' deep understanding of the prevailing and underlying mathematical theory and concepts and not one based on the visual, tangible or purely *aesthetic*; and as such I fear this level of understanding of the *intellectual beauty* of the equation as a conceptual work in itself is going to be far beyond the understanding of most and no doubt only truly recognisable to those with the extraordinary ability to work with these abstract mathematical notions.

To quote a very contemporary reference that precisely mirrors my view but from a mathematician's perspective, June Huh, Professor of mathematics, Princeton and winner of the Breakthrough Prize in Mathematics, 2019, said in his video interview titled *"What motivates us is the pursuit of beauty"* (Huh, 2019) "as mathematicians we often do not have anything in mind when we are constructing our theories …. what motivates us most is the internal beauty of the (maths) we create". He goes on to discuss the often quoted question of whether mathematicians are *creating* something or simply *finding* existing states and ideas that are, and always have been, out there in the universe to be *discovered*. He continues by saying that " .. I believe the motivation is very similar to that of artists." I find it fascinating hearing almost my exact words, albeit from an arguably disciplinary *polar opposite* position, being spoken by such a noted mathematician; this is clearly a subject that vexes practitioners on either side of what I refer to as the *logic divide*. He also advances an idea describing the method a mathematician works through, which is naturally, logic based and can be seen as a progression of sentences, where sentence number two follows on logically from sentence number one and so on; however he also says mathematicians are often really surprised when they arrive at say, sentence one hundred and find that it bears no relation to the *opening gambit* of the equation. In other words the equational methodology used can take a mathematician on a journey into their very own *beyond;* to arrive at a conclusion that they could never have predicted or expected when formulating the very first line of numbers, symbols and words. To use Huh's own words he describes this unexpected result rather poetically himself as *"… something that goes beyond our naïve intuition"* (Huh, 2017).

I believe it is this ability to allow oneself to be sufficiently open-minded, to always embrace, nurture and *listen to* one's own *naïve intuition* that allows artists to push their ideas and their art into areas that will reveal something new to them, something that is currently an *unknown.*

The attraction of the unknown and the *unknowable*

We now inevitably arrive at the question I feel this essay is really posing; and that is "what can an artist possibly get from a *seemingly* impenetrable abstract mathematical text?". Or, perhaps even more pertinently, what do *I* get from it? As I am not qualified to answer the question on behalf of any artist other than myself, this interpretation and reading of the subject can only ever be that - purely *subjective* and, naturally, *personal*. I think however we can draw inference from, and perhaps gain an insight into, other artists' motivations and inspirations and make the link back to some more obvious mathematical or scientific influences by studying the clear references in their practice too. I am using mathematics and science here as plausibly interchangeable disciplines of inspiration for artists; the one, science, being *largely* informed and underwritten by the other, mathematics; the subjects are intellectual bedfellows in the broadest and most fundamental of terms. The fact that so many artists over the years have either been inspired by mathematics, whether pure or applied (to science), makes it clear to me that I am very far from alone in having this fascination for what I am sure is, to most contemporary artists, *the unknowable*. A very high profile example would perhaps be Damien Hirst's appropriation of chemical compound names for the titles of the body of work that subsequently became known as the *Pharmaceutical Paintings*; canvases consisting of random coloured spots painted in geometric grid pattern, by hand, to look immaculately and almost *scientifically conceived;* this affords the works a *pseudo-scientific* quality that I am sure was intended to raise questions of order, classification and taxonomy and is definitely inspired by the inherent unknowable *scienceness* of the words themselves; which I feel makes Hirst's recognition of the poetics in these words much the same as my finding the language of topological equations and theories inspirational. Today we also find some of the most avant-garde contemporary art practitioners working at a molecular level with computers, fractals, electron paths, coding and *Artificial Intelligence* (AI) in general, pushing the boundaries of the technology to embrace and actually *embed* the mathematics deep within the art itself; for the numbers *per se* to actually *become* the art. We can see this very clearly in the work of German artist Mario Klingemann, who is perhaps most famously-known for creating the first AI artwork to ever be sold in a *traditional* art auction house[5], *'Memories of Passersby I' (fig.7)*. In this work he uses neural networks of code to generate portraits on two LCD screens hung on the wall above a traditional piece of furniture that houses the computer system. For Klingemann the art is not the resulting images but the *algorithmic code itself*; the mathematics, the numbers and their function *are* the art.

Fig.7 'Memories of passersby I' 2018 by Mario Klingemann

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[5] *'Memories of Passersby I' sold for GBP £40,000 at Sotheby's Auction House, London, 6 March 2019.*

So, do we deduce from this that Klingemann is the artist who created the work; or is the computer the artist? Klingemann clearly wrote the *original* code but the computer is the 'brain' that actually made the *decisions* that *dictated* how the portraits would look and not Klingemann; the artist created the environment and the opportunity for the computer to *do its own thing,* to make art*.* However, I feel in the case of *'Memories of Passersby I'* it is clear that there is more to the work than *solely* the AI. The AI, the code, the *maths*, are all just *part of the art,* the installation*,* and there is much in the content and presentation that is obviously directly attributable to Klingemann as the artist and nothing to do with the computer; the choice of cabinet, frames and practical specified arrangement of the elements that make the piece as a whole. Perhaps in light of this thinking it would be more accurate (and no doubt controversial) to describe the artwork as a *collaboration* between Klingemann *and* the computer? These philosophical questions regarding the intellectual *propriety* of the art that gets made using AI and coding techniques will, I am sure, be coming more and more to the fore as topics of debate as this cutting-edge technology is adopted and explored by the new-wave of artists pushing at the doors of our very own perception of what is and what is not *art.*

And so, returning to my question at the top of this section *"What do I get from the reading of abstract mathematical text?"*, my purest answer to this can be summed up most directly as *language, poetics and metaphysical imagery.* To make the assumption that all mathematics is simply *numbers* is a gross misconception, and Dowker's '*Lectures on Sheaf Theory'* (1952) text, the original motivation behind this investigation, is anything but *just* numbers; it is a beautifully-crafted *typographic* narrative, composed of a strange esoteric language, illuminated by symbols indecipherable to the *unknowing* eye, which encourages, at the same time, feelings of wonder and confusion similar to that no doubt experienced by the earliest archaeologists facing the wall of an ancient Egyptian tomb covered in hieroglyphs for the first time. It is an entire *other* language, a *code*, known and understood only by a knowledgeable, and in the case of mathematics, privileged and highly intelligent, educated few; the *majority* being excluded from its meaning as we are not privy to the knowledge needed to decipher and make sense of this language - it is so different, so difficult, so *alien*. When one scans through a mathematical thesis as Dowker's it is clear, by intent, that it contains and describes a *narrative, a path, a point-argued.* There is very obviously a beginning and an end; there is a first line and there is a last line, and the text in between is a magical, meandering, metaphysical journey weaving its way through a world of its own creation and imagining, with waypoints, markers and semiotic signposts; if only one could understand? Just a brief glance through a few flicked-pages will reward investigation with such words as - *homomorph, isomorph, abelian group, euclidian, paracompact, homeomorphism, sheaf space, twisted integers, sub sheaves and so on.* These peculiar, often beautifully-sounding, *alien* words I find highly suggestive of shape, form, space; *conceptual* space inhabited by strangely-named *beings*? These otherworldly words take me on journeys; internalised creative journeys that are not easy to describe and that I don't even fully understand myself when I am taking them. An *isomorph*, to take one example, to me suggests a *thing*, a *being*, something that could even be described as alive, as a *real thing;* something one could touch and feel; it could be totally *other or extra-sensory* rather than manifest itself as a solid form. It suggests an ethereal presence, an intangible *anamorphic* shape, a pliable, flexible, shape-shifting form, defying physical explanation and design. These are the poetical avenues and alleyways along which I find myself purposefully getting lost in, *sans GPS or psychogeographic* help, while imbibing this most opaque of abstract, algebraic mathematical text; a blind and inquisitive wanderer touching, feeling and almost *smelling* my way around.

Knot theory, being the other major branch-study within this alien *topological landscape* we have considered here, also demands and arouses my artistic attention and investigation in equally fascinating ways. Distinct from the *unimaginable* conceptual spaces of Sheaf Theory, Knot Theory has its roots firmly planted in that which is actually *visualisable*; one can actually take a length of wire and model a knot in three dimensions and *touch* it, look at it from any angle. But I feel that applying this prosaic thinking is to minimise the beauty and complexity of knots and certainly fails to recognise the myriad ways in which knots occur within nature and can touch our own lives. The modelling of DNA structures uses Knot Theory to untangle and describe these complex, beautiful forms into computational equations that scientists can use to understand the very building blocks of life itself. To my mind the notion of complex knots is readily suggestive of life itself; of relationships and connections and the complexities of communication. A knot can be visually representative of a very personal journey or feeling; of quite literally *tangled* emotions between people in life's constant-flux dynamic.

By way of a notional and highly subjective conclusion, my own work and investigations into these uncharted areas, using what little I understand of the mathematics, inspires me to continue to embark upon these journeys into the currently *unknown* much as the mathematicians we have looked at here start out on their own personal *equationally-based* journeys; neither knowing what the final result will reveal. That art and mathematics will continue to be fascinated by *each other* is, I believe, beyond doubt. That the integration of deep-code, computational mathematics into all areas of investigation will continue to inspire artists and mathematicians alike, in the production of newer and currently *unknowable* work outcomes is inevitable. One can't help but wonder how much more abstract and notional artists' work inspired by, and using, mathematics could become? Would the ultimate distillation of this occasionally awkward academic relationship end (or perhaps *begin?*) with an artist proposing solely *the numbers* as an artwork in themselves, bereft of any *physical* manifestation? Could one ever propose a naked *equation or code* as art; offered for sale (as surely all art is ultimately commodified) in its unadorned numerical form, and presented *exactly* as written by a mathematician, and could this be labeled and repurposed as a '*mathematical readymade'*? One suspects that should Duchamp be alive today this idea would certainly not be beyond the unthinkable.

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